Team Theme (Math Challenge 2007)
Growth/Decay – How Do I Rate?

INTRODUCTION: Measuring growth and growth rates is an important activity in business, industry, social sciences, and many other areas. Of course, mathematics is an essential component in this activity as we explore and discover ways to measure growth and decay.

A. This is a growth example that interests Carla who has completed her degree and is looking for a job. She has received the following job offers from two different companies. Company A is offering Carla a beginning hourly rate of $12.40 and promises to give an increase of $0.30 (30 cents) per hour every three months over a four year period. Company B is offering Carla a beginning hourly rate of $10.10 and promises to give a 5% increase in her hourly rate every four months over a four year period. Carla must decide which job to take. The working conditions, responsibilities, and benefits are very similar for the two jobs. Here are some questions Carla would like answered before she makes a decision. Can you answer them for Carla? (Show or explain carefully how you arrived at the answer to each of Carla's questions.)

    1. At the <u>end</u> of the 4 year period, which company will be paying the higher hourly wage? Include amounts for each company.

    2. At the end of the 4 year period, at which company would Carla have earned the most money? (Assume that Carla worked an average of 150 hours per month over the 4 year period.)

    3. Mathematics has names for the two different growth patterns as illustrated by the increases in hourly wages given by Company A and Company B. The hourly wage increases given by Company A is called an arithmetic progression and the hourly wage increases given by Company B is called a geometric progression. Explain the difference between an arithmetic progression and a geometric progression.

    4. To facilitate the computation done in A-1 above, mathematicians look for growth patterns in each case and try to derive a formula which summarizes the results. These formulas can then be used to do rapid calculations when different input is used. a) For Company A let the beginning hourly wage be "a", let the increase given every 3 months be "d", and let the number of increases given over a period of time be "n". Find a formula (in terms of a, d, and n) which would find the amount of the hourly wage after "n" increases have taken place. b) Find a formula for Company B where the beginning hourly wage is "a", the rate of increase is "r", and the number of increases over a given time period is "n". (Note: If a 5% raise is given, then the rate of increase "r" will be 1.05.) Test your formulas by using them to find the answers in part 1 above.

B. Populations can also grow as a geometric progression.   Let's see what this means for a colony of bacteria <u>in which the population doubles every minute</u>. *Suppose that at 12:00 noon a single bacterium colonizes a discarded food can. The bacterium and his descendents are all happy, but they fear "Doomsday" – the time when the can is completely full of bacteria and has no room for more population.* Answer the following questions about this bacteria colony.

1. Do you know of another name to describe the growth of this bacteria population? If so, what is it?

2. How many bacteria are in the can at 12:05 pm?   At 12:10 pm?

3. The can is completely full of bacteria at 1:00 pm. At what time was the can only half full of bacteria?

4. When the can was exactly half full, the president of the bacteria colony assured his constituents that doomsday was still far away – after all, he reasoned, there is as much room left in the can as has been used in the entire history of the colony. If your group were the president's special advisory council, what advice would you give him?

5. When the can is one-quarter full, how much time remains till "Doomsday"?

6. The resident mathematician in the bacteria colony advises that they start a new colony beginning with 4 bacteria in a can of the same size as the one they are now living in. She also recommends slowing the growth rate to a doubling of the population every 2 minutes instead of every 1 minute. How much time would this new colony have until "Doomsday"?

C. The following is a *New York Times* article written in 2006 about population growth in America.

<u>*Let's make history, baby*</u>
<u>300 millionth American is about to be conceived, experts say</u>

THE NEW YORK TIMES

If the experts are right, some time this month, a couple will conceive a baby who will become the 300 millionth American. As of Thursday, the Census Bureau officially pegged the resident population of the United States at closing in on 297,900,000.
The bureau estimates that, with a baby being born every 8 seconds, someone dying every 12 seconds, and the nation gaining an immigrant every 31 seconds on the average, the population is growing by one person every 14 seconds.
At that rate the total is expected to top 300 million late this year.

The conclusion of this article based on the information given is that: "… *the population is growing on an average of one person every 14 seconds.*"   1) Explain in detail how they might have come up with this conclusion. 2)   If the article was written on May 4, 2006, and their calculations are correct, how many **<u>days</u>** would it take for the population in to reach 300,000,000?

D. All of the above examples that you worked through were samples of <u>growth</u> taking place. Make up a realistic example of decay or population decline taking place. Use actual numbers in your example and predict a future amount or future population in a given time period of your choice. Indicate whether your example is a declining arithmetic progression, geometric progression, or neither. (Points will be awarded based on content, correctness, and level of sophistication.)