

# Math Challenge

2010

## Team Theme

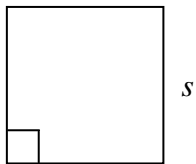
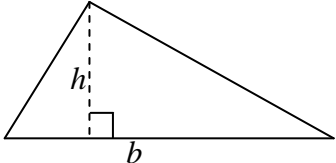
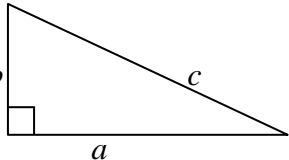
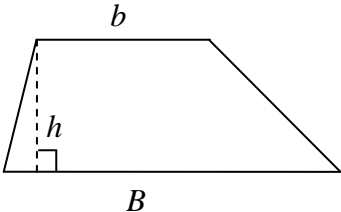
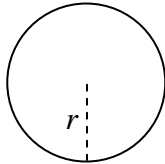
**This is a TEAM event. You are to work on the Team Theme as a group. Please use the solution sheets to write your final solutions, and be sure to put your TEAM NUMBER on each page (your team number is listed on the back of your schedule sheet). Do not use your school name.**

**If you need extra space to answer a particular problem, you may use the backside of the solution sheets or attach more than one copy of the solution sheets (each team has been given 3 copies). If you do use a backside or attach another copy of the solution sheets, please make a note so the grader knows where to look.**

**Write all answers in mathematically and grammatically complete sentences. You will be graded not only on the accuracy of your answers, but also on how well your team explains itself mathematically, and on the creativity in your narrative. It is your job to convince the graders that you understand the problem completely and to make your presentation interesting.**

Throughout your mathematics courses you learned how to compute the areas of certain geometric figures using special formulas. In this team theme you will be asked to extend your area concepts to some unusually shaped figures and perhaps discover some “new” formulas for finding areas.

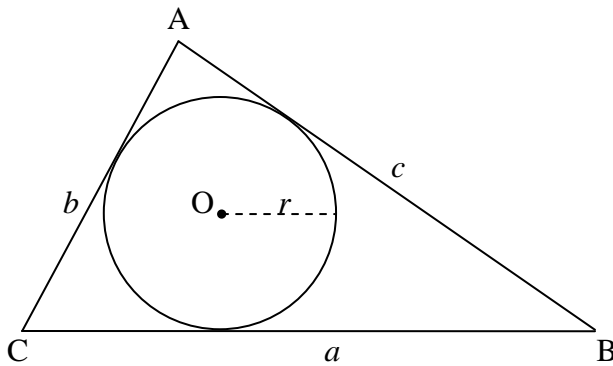
1. First, make a list of area formulas you will need later. Write the formula for each of the following familiar geometric figures using the variables in the drawing.

	Figure	Drawing	Area
(A)	Square		A =
(B)	Triangle		A =
(C)	Right Triangle		A =
(D)	Trapezoid		A =
(E)	Circle		A =

Now it's time to check how combinations of formulas in #1 can lead to new formulas and solve new problems.

2. If the radius of a circle is increased by 50%, by what % is its area increased?

3. Circle  $O$  is inscribed in triangle  $ABC$ . Find a formula for the area of  $\triangle ABC$  using the radius  $r$  of the circle and the perimeter  $p$  of the triangle. Explain clearly how you obtained your answer.



4. In the figures below, figure 1 shows a square inscribed in a semicircle of radius  $r$ . Figure 2 shows a square inscribed in a circle of radius  $r$ . Determine the ratio between the area of the square inscribed in the semicircle and the area of the square inscribed in a circle. Explain clearly how you arrived at your answer.

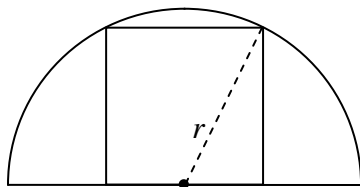


Figure 1

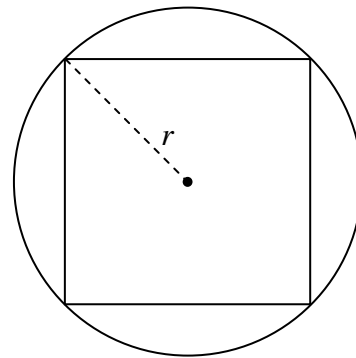
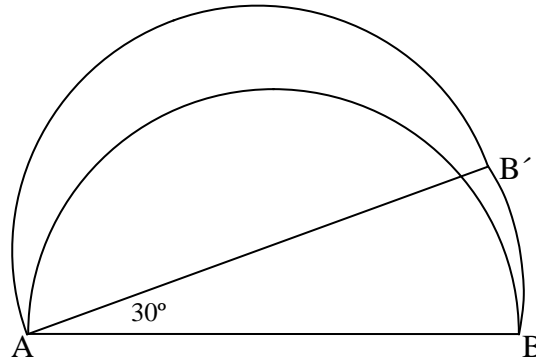
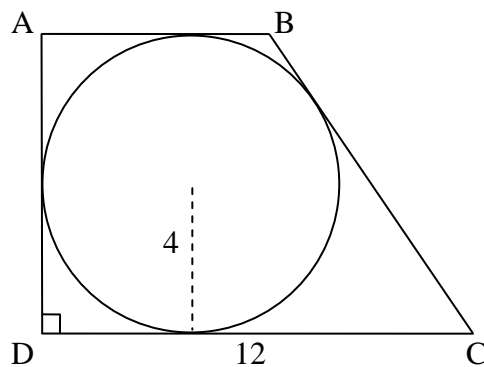


Figure 2

5. In the figure below, a semicircle with diameter  $AB$  with the length 10 cm is rotated counterclockwise by  $30^\circ$  about the point  $A$ . As the point  $B$  is moved to  $B'$ , the arc  $BB'$  is traced. Find the area of the shaded region. Give exact answer including units and an approximated answer rounded to 3 decimal places.

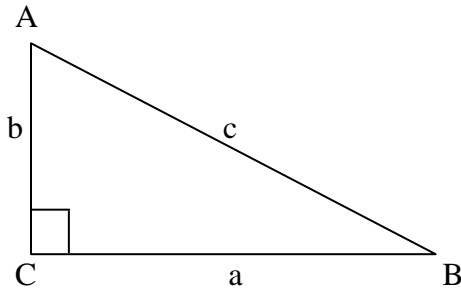


6. In the figure below, a circle of radius 4 inches is inscribed in a trapezoid  $ABCD$  whose longer base is 12 inches. Find the area of the trapezoid. Give exact answer including units. Explain clearly how you arrived at your answer.



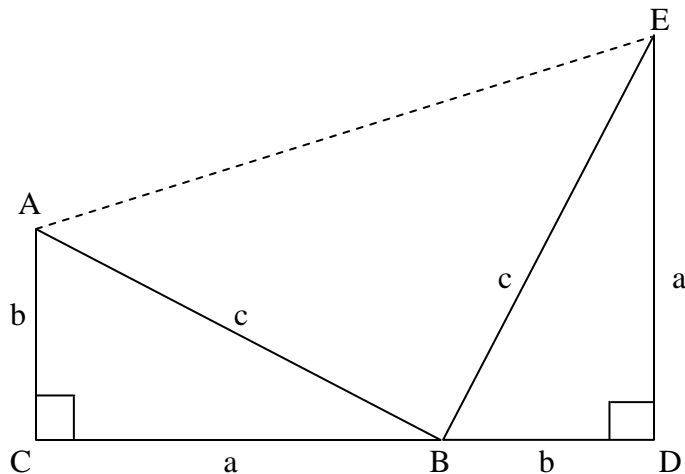
The Pythagorean theorem states that “*In a right triangle, the sum of squares of the legs is equal to the square of the hypotenuse.*”

**7(a). Given a right triangle ABC below, state the Pythagorean theorem in algebraic terms.**



There are many different ways to prove the Pythagorean theorem. President James Garfield (1831 – 1881) used areas to prove the Pythagorean theorem.

**7(b). Using the figure below, explain clearly how president Garfield proved the Pythagorean theorem.**

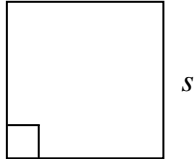
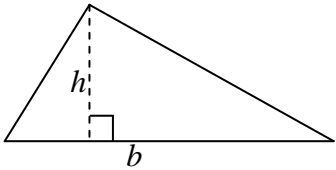
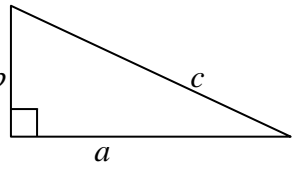
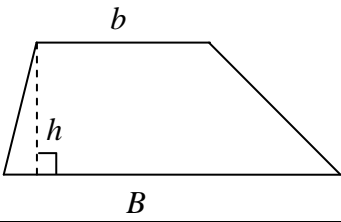
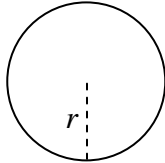


**8. Compose a problem of your own related to area of geometric figure(s). Then give a solution to your own problem. Explain clearly how you arrived at your answer.**

# Solution Sheet

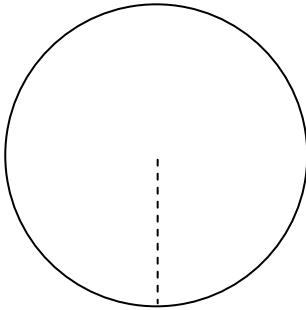
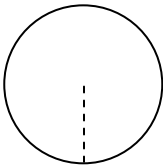
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1.

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(B)	Triangle		A =
(C)	Right Triangle		A =
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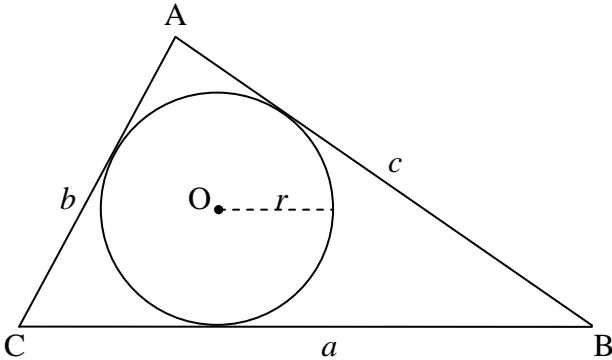
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2.



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3.





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4.

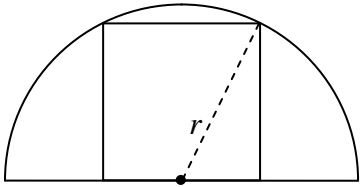


Figure 1

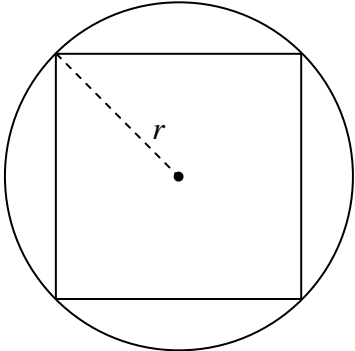
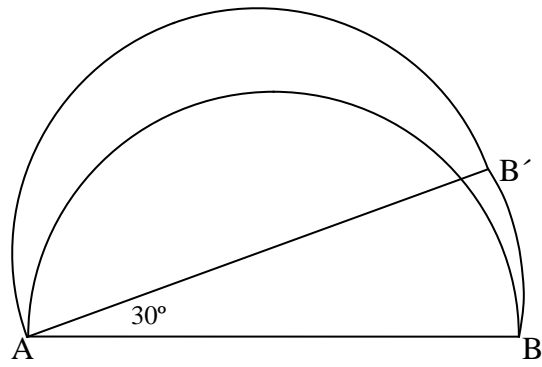


Figure 2

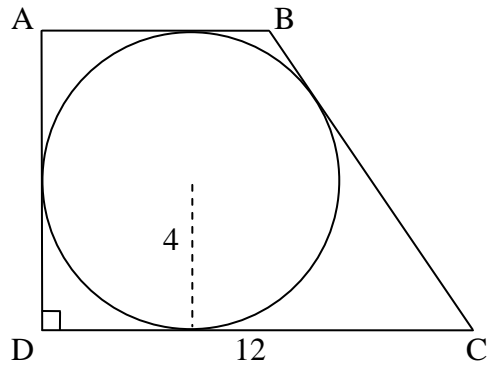
5.

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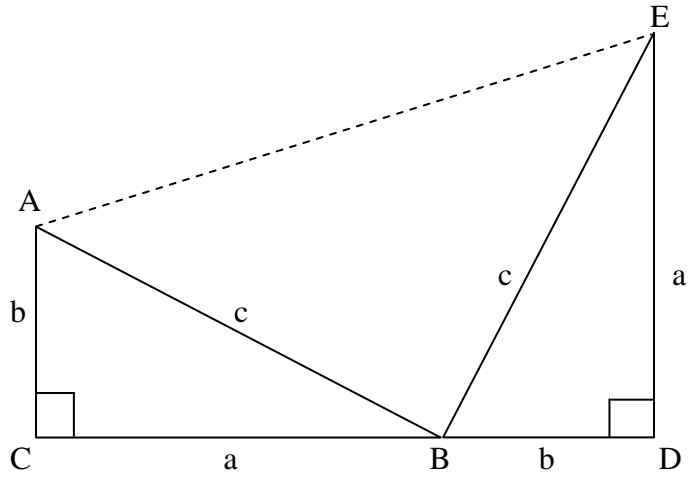
6.



Team #: \_\_\_\_\_

7(a).

7(b).



- 8. Compose a problem of your own related to area of geometric figure(s). Then give a solution to your own problem. Explain clearly how you arrived at your answer.**